TELECOMMAND AND NAVIGATION

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1. INTRODUCTION

Since the days of the Babylonians, through the time of the Greek and Roman Empires, man has displayed an ever-growing need for accurate mapping of the Earth's surface and for means of finding his way about on it. In particular navigation has been the problem for mariners over the centuries, although it was only in comparatively recent historical times that navigational requirements encouraged the invention of the compass, the sextant and the chronometer. From then onwards, dead-reckoning, based on time and vector velocity measurement, with fixes on sun or stars, has been the basis of most methods of surface navigation.

In the first days of powered flight, navigators used the original method of the early mariners—visual observation of land. Later, while speeds were still low and flying was confined to good weather, a simple airspeed indicator, magnetic compass and manual course plotting proved adequate; but advances in aircraft performance and range soon called for improved dead-reckoning equipment and the development of new fixing aids. During the last war automatic D.R. was developed and integrated with airborne radar fixing devices. Numerous radio and radar navigation aids appeared, such as Gee(1) and Decca(2,3) for general purpose position fixing, or Gee H and Oboe(4) for specialized tasks such as the laying down of precision tracks for blind bombing. Airborne radar gave defensive fighters the ability to locate and home on to hostile bombers, while automatic approach and landing aids were based on the use of radio beams and radar beacon systems. In all these systems an important function was performed by the pilot or navigator who applied human intelligence to monitor the functioning of the airborne navigation equipment and filtered the various types of information presented by the basic instruments so as to combine in the best possible way the data available.

In the case of the unmanned flight vehicle, the absence of human intelligence dictated a more sophisticated technical approach to the problems of navigation and guidance. Thus, work in the United Kingdom in the 1920s on unmanned bombers and target aircraft resulted in the development of fully automatic flight with telecommand and the beginnings of automatic D.R. navigation based on the air log and compass-monitored course keeping. It is interesting to note that this simple navigation system re-appeared some 20 years later in the German V.1 flying bomb. Towards the end of the war the forerunners of today's guided missiles began to emerge and in one of these, the German V.2, the application of an important new technique of D.R. navigation—inertial navigation—was heralded with the use of an integrating gyro-accelerometer. This technique offers many desirable features; it is self-contained and non-radiating; it is capable of providing navigation data in three dimensions; it can provide high quality short period information on acceleration, velocity and position which can be of great importance in automatic guidance systems. In common with other forms of D.R. navigation, if high quality long period information is needed, great demands are placed on the performance of the vital components—gyroscopes, accelerometers and integrators—on which the technique is based.

Another important new technique, radio Doppler[(5)], came out of work in the aircraft field towards the end of the war. Its development has proceeded apace and it is now widely used in aircraft. Doppler gives for the first time a direct precise measure of groundspeed for use in D.R. navigation systems. Precise direction is provided also but unfortunately related to aircraft axes; definition of these axes relative to the ground awaits a solution of similar elegance to that for groundspeed.

Since the war the development of navigation and guidance equipment for the manned aircraft and guided missile have proceeded side by side, and in many cases the application of techniques developed initially in the one case can be seen applied to the other. Thus, the tremendous strides made in radar developments for manned aircraft during the war permitted the application of relatively advanced radar techniques to the early guided missiles. A missile with an active radar homing head can reasonably be likened to a very high performance pilotless fighter with an automatic A.I. (airborne radar interception) guidance system. Even in the manned fighter, very high performance and high speed targets pose problems in which the time element needed to make full use of human appraisal is critical and more nearly automatic navigation systems are inevitable.

The supersonic anti-aircraft missile, the ballistic missile and the Earth satellite rocket are all deprived of the human intelligence which might filter the various types of navigational information fed to the missile automatic guidance and control system so as to maximize the accuracy with which the desired path is followed. In addition, the high speeds demand extremely rapid response. From the beginning, therefore, such missiles have had fully automatic systems of navigation, guidance and control, and great emphasis has been laid on the development of methods for reducing errors due to imperfections in the basic position or velocity finding devices or in the missile control system or the guidance computer. At the same time much attention has been given to methods of combining different basic navigation techniques to produce a result more accurate than can be achieved using any one technique alone.

In this paper the principles underlying various types of automatic navigation and guidance systems are described, and examples of one or two specially interesting developments are given. Since the unmanned vehicle has, as mentioned above, demanded the development of advanced and elegant techniques of navigation and telecommand, attention will be concentrated rather on missile problems. The accent will, however, be on the underlying principles rather than specific developments. A very special problem is posed by the need for guidance towards a moving target and so the anti-aircraft missile is chosen to highlight some of the techniques used as a basis for advanced missile systems. The final section of the paper discusses briefly some of the navigation problems of satellite space stations.

2. GUIDANCE FOR THE ANTI-AIRCRAFT MISSILE

2.1 *Design Principles*

The various well-known forms of anti-aircraft missile guidance, by command, beam riding or homing, essentially follow a single pattern in

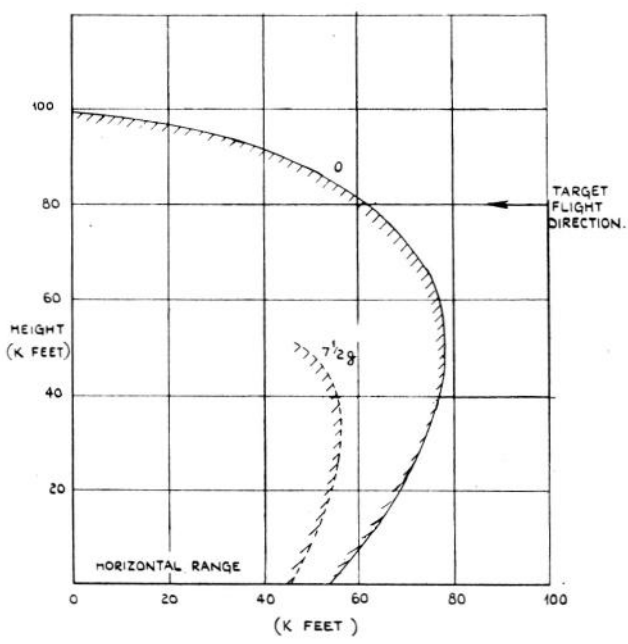


FIG. 1. Loss of range due to induced drag for typical beam-riding missile. Curves show maximum range against constant speed target flying at constant height. Values on curves denote root mean square acceleration available to cope with target evasion.

using a device to gather information on the target and to convert this information into a form suitable for control of the missile by the application of aerodynamic forces. The subsequent motion of the missile modifies either the information being gathered or its interpretation so that the whole system operates as a servo-mechanism of considerable complexity attempting to reduce to a minimum the separation of target and missile. It is common practice to devise navigation systems for the missile to give as near as possible a collision course interception, although other courses such as pursuit and beam riding may have advantages under certain conditions.

The main reason for realizing successful interception with guided missiles is that the missile is normally capable of a high lateral acceleration compared with the target and, in addition, its response to demands is faster. This assumes that the missile possesses either a speed advantage or operates over a limited sector with respect to the track of the target. Immediately it is evident that the provision of high lateral acceleration imposes structural problems on the missile and, if employed frequently, will reduce the range and speed performance due to additional induced drag (Fig. 1). Thus on grounds of economy and performance it is important to design the overall guidance system to achieve interceptions with the minimum use of lateral acceleration. One criterion can be the achievement of a given r.m.s. miss distance with minimum lateral acceleration demands averaged over all possible engagements, suitably weighted according to their frequency of occurrence.

The requirements on the missile depend on the properties of the target, such as speed, height and manoeuvrability (lateral acceleration). In addition, the information gathering device will provide signals subject to random fluctuations or noise due to the nature of the target. For example, the radar reflections from an aircraft are subject to fluctuations in amplitude and in their direction of arrival. These fluctuations are due to the nature of the reflecting object and its motion which have the effect that surfaces of constant amplitude and of constant phase are not spherical. In addition the form of these surfaces changes at some point within the Rayleigh distance from the target, the precise point depending on the effective aperture of the target considered as a radiating source.

From the point of view of the information gathering device no distinction can be made between the wanted signal which gives a measure of target maneouvre and target noise. In addition the information gathering device will be subject to its own internal noise and further noise will arise in the missile control system. These sources of noise are largely under the designer's control but, beyond a certain stage, it becomes uneconomic or impossible to reduce them further. Difficulties can arise when part of the control servo-mechanism noise is a function of the signal since the use of filters may not be effective in reducing it. In practice such noise can be largely eliminated by careful design.

If statistical information is available on the manoeuvre capabilities of the target and on the noise then it is possible to distinguish partially between them. Thus if the frequency spectra of signal and noise are in separate frequency bands then complete distinction can be obtained using a band pass filter. If, however, the frequency spectra are identical then there is no means of distinguishing one from the other without more detailed knowledge of how either signal or noise vary with time during a particular engagement. In practice it is usual for the spectra of signal and noise to overlap and the problem is resolved into finding the best filter to maximize the signal to noise ratio. This is a problem which the communications engineer has studied in considerable detail and provided solutions for certain classes of problem. The statistical methods developed by Wiener(6) can be applied directly to guided missile systems which are linear, i.e. those which can be represented mathematically by linear differential equations. In practice such systems do not exist but considerable advances have been made in choosing approximations to linearize the equations so that the theory is applicable over part of the interception. Such methods have had a profound impact on anti-aircraft guided missile design since the choice of a suitable filter can alter markedly the r.m.s. miss distance and the lateral acceleration requirements, without appreciably increasing the number of components or affecting the reliability of the system. However, due to the non-linearity of real systems it is not sufficient to apply the theory directly and the following are typical of the methods employed to find the optimum system design.

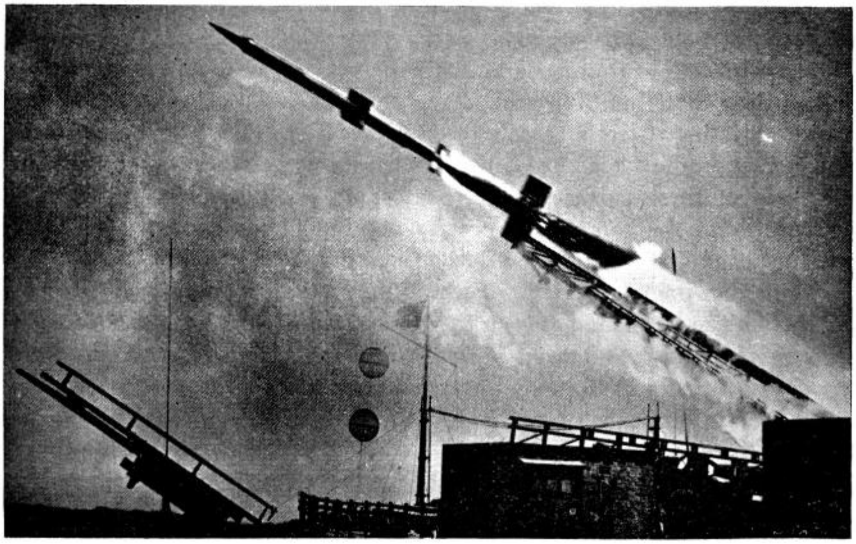


FIG. 2. Beam-riding test vehicle leaving launcher.

(1) System study to make suitable approximations and determine the best transfer function for the missile system, knowing the design constraints likely to be met in practice.

(2) Practical realization of a suitable filter to give this transfer function

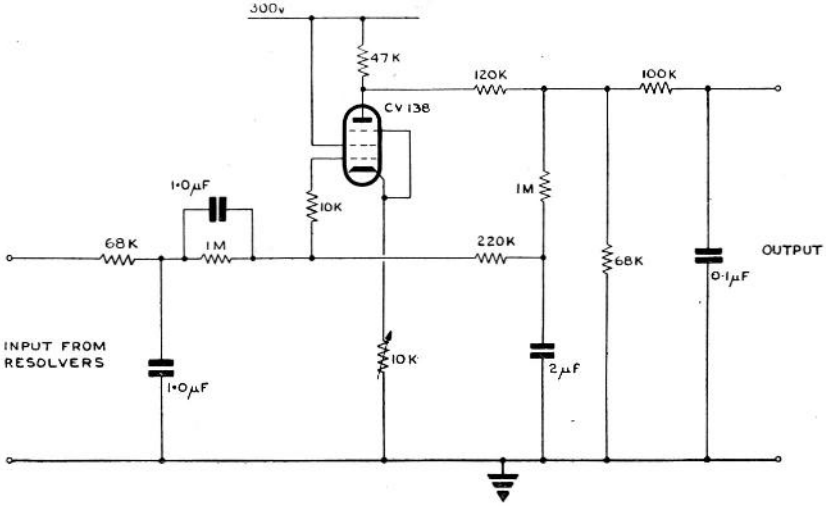


FIG. 3. Beam-riding system. Early phase-advance filter.

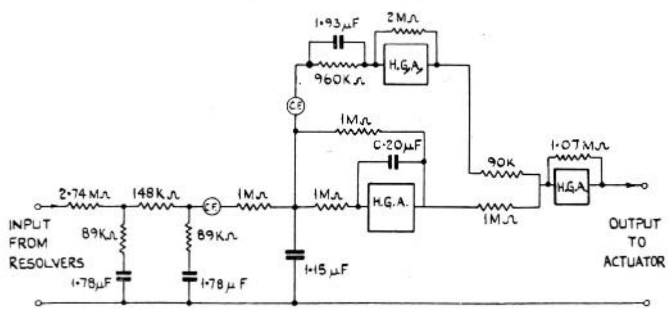


FIG. 4. Beam-riding system. Active optimal filter for random target acceleration.

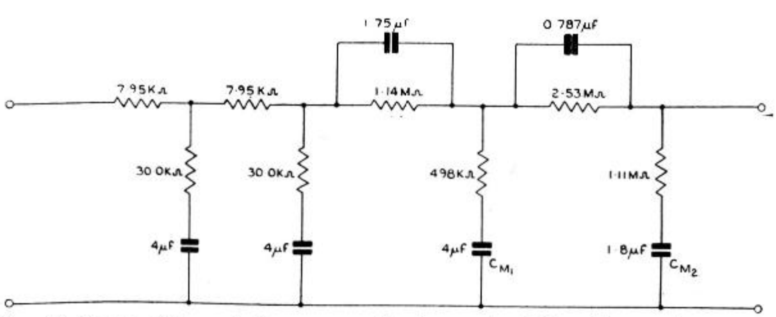


FIG. 5. Beam-riding missile system. Passive optimal filter for random target acceleration.

to the system. In practice it is often possible to find a passive filter of adequate performance.

(3) Check of the system for a large number of interceptions using simulators (analogue computers). Non-linearities are included in the simulator study. A comparison can be made between a simple stabilizing filter and the theoretically optimum filter. A few checks are made using digital computers to confirm or otherwise the accuracy of the analogue computation.

(4) Flight trials to provide particular verifications of the simulator work.

Each of these operations calls for special types of knowledge and experience and the results interact on one another. Thus the results of the simulator indicate modifications to the analytical treatment and results of flight trials indicate modifications to the simulator programme. An example of this work, carried out several years ago at the Royal Aircraft Establishment for a beam-riding test vehicle is given below.

2.2 *Optimization of a Beam Rider*

The test vehicle is a cruciform fixed wing missile with liquid fuel sustainer motor and tandem boost (Fig. 2). In the early design of a beam riding system a simple phase advance network was included to ensure stability in the guidance loop. This proved satisfactory for carrying out early trials on beam riding systems. It was realised, however, that it did not result in the optimum transfer function for the missile system. A study of the problem was made using the following assumption about target manoeuvre. The target aircraft should be capable of lateral accelerations up to $2g$ and the pilot would demand these accelerations in a random manner in time and with a random value of amplitude. Under these conditions what is the optimum transfer function of the missile system and how can such a transfer function be realized in practice?

Using the methods of Wiener and approximations to linearize the system the required filter was determined. This was an active filter; a passive filter was also designed. Three filters, a simple phase advance, an active filter and a passive filter are illustrated in Figs. 3–5. These filters were used in simulator runs, which showed that considerable reductions in miss distance were obtained by replacing the simple phase advance filter by either of the others. There was nothing to choose between the active and passive filters and therefore the latter was used in flight trials.

One of these trials was carried out in a simple manner. The beam of a radar was programmed with jitter (noise) corresponding to that likely to be obtained when tracking a real target. At a given time during flight of the missile an additional programme, corresponding to a $2g$ turn of the target, was applied. The requirements for lateral acceleration and the resulting miss distance agreed closely with the simulator results and showed the advantages of the new filter design in a striking manner. The results of the simulator and flight comparison are given in Fig. 6.

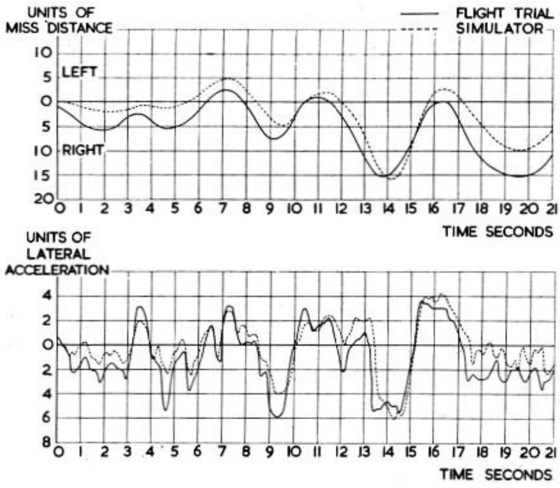


FIG. 6. Beam-riding in the presence of radar noise and target manoeuvre. A comparison between flight trial and simulator results.

2.3 *Optimization of Missile Systems against Target Manoeuvre*

This early optimization of the missile system indicated the importance of scientific system study as well as advances in component technology. The system was, however, optimized in a particular class of circumstance, namely when the target applies lateral acceleration in a random manner. If the opponent knows that the defensive missile has been designed in this way, he can apply a manoeuvre which maximizes the miss distance, the actual manoeuvre depending on his knowledge of the missile systems and the whereabouts of the defending missile. Under these circumstances, the filter which is optimal against random acceleration may be less beneficial than the simple phase advance filter; this is because the attacker is using a special manoeuvre to defeat the particular filter. Corresponding to any particular missile transfer function, there is a worst target manoeuvre such that any departure from this manoeuvre decreases the miss distance. Provided it is assumed that the necessary knowledge is available to both sides then, restricting to linear systems, we obtain by theory of games methods a stable known situation covering an interception. Under these circumstances, the opponent needs only to demonstrate his capability of lateral manoeuvre: then, instead of being able to smooth heavily the noise in the system on the presumption of a constant velocity target, the defender must arrange to detect lateral manoeuvre as rapidly as possible. This immediately increases the sensitivity to high frequency noise and so the average miss distance even against a non-manoeuvring target. Once this has been done, the attacker loses little by flying straight and level to the target since the defender has been forced to design his missile system on the basis of manoeuvring target capabilities.

Evidently there is a conflict between designing for the most probable conditions and the worst possible eventuality. The defending missile system should be able to deal very effectively with a target flying straight and level since this is the most likely condition. This would result in poor performance against a manoeuvring target. The obvious solution has been a missile system capable of learning from the target and adjusting its parameters to the particular circumstances. This has led to the use of sympathetic servo-mechanisms which self-optimize the system. The theory of such systems is complex, since by their very nature the systems are non-linear during the process of altering their parameters. Certain simple examples of self-optimizing have been described, in particular, the example given by Burt[7].

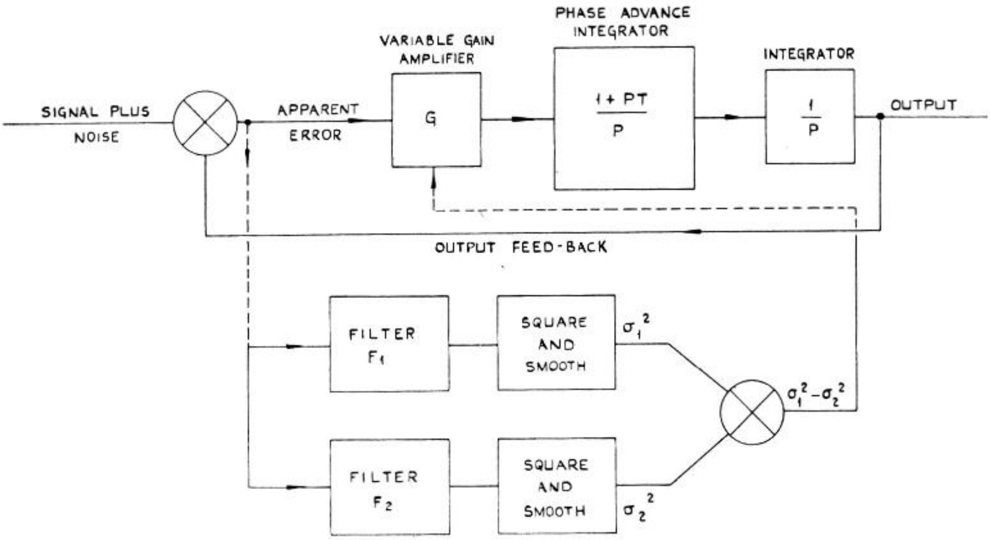


FIG. 7. Feedback noise filter with optimal gain controller connected by broken line.

2.4 *Self-optimization*

In this simple example, attention is concentrated on variations of target manoeuvre and noise which vary in their mean square values but leave the shapes of the frequency spectra unaltered. Suppose, for illustration, that the signal spectral density is K^2/ω^4, so that the signal acceleration behaves statistically like white noise. Here K is a constant and ω is the angular frequency considered. Suppose also that the noise spectral density is k^2, where k is constant, so that the noise is white. If this signal plus noise is regarded as the input to a typical servo, such as that indicated in Fig. 7, it can be shown that the apparent error (i.e. the difference between the output and the signal plus noise) has a spectral density given by

$$k^2 \; \frac{\omega^4+K^2/k^2}{\omega^4+G^2}$$

where G is the loop gain and the time constant of the integrator is chosen to be

$$T=\sqrt{\frac{2}{G}}$$

If the gain is chosen to be $G=K/k$, the error spectral density is constant with frequency, which can be shown to be the condition for an optimal system in the presence of a white noise. The mean squared error is equal to $4K^{\frac{1}{2}}k^{3/2}$.

In practice, adjusting the loop gain to the optimal value of K/k is not easy since the values of K and k are not separately available. However, the constancy of the error spectral density can be used as a criterion for obtaining the optimal system. It will be seen that if $G>K/k$, the spectral density increases with frequency, while if $G<K/k$, it decreases. Thus comparing the power outputs of two filters fed by the error signal indicates whether the loop gain G should be increased or decreased. These two filters must, of course, cover different parts of the spectrum, although an overlap does not matter. If their bandwidths are equal, the loop gain G equals the optimal value K/k when the mean squared outputs are equal. The difference between the mean square outputs measures the magnitude and sign of the error in gain and can be used in an auxiliary servo loop to maintain $G=K/k$.

Although the above discussion has been based on the assumption of stationary time series, a similar analysis will apply if the signal and noise powers K and k change slowly. Provided the spectra can be considered quasi-stationary with constant shape and slowly varying powers, the gain adjustment will operate to maintain the system in an optimal condition.

The mean squared outputs of the pair of filters are subject to smoothing over some time interval. The value of this time constant is important. Evidently it should be short to keep the system as near as possible to its optimal condition: on the other hand, a very short time constant could lead to instability. Analysis of the system is difficult since, although the main loop is linear, the addition of the auxiliary loop renders the whole system non-linear; but some guide to the general behaviour of self-optimizing systems can be obtained from analogue computer studies.

2.5 *Some Problems of Practical Anti-aircraft Guidance Systems*

A guided missile should function under all conditions against the target for which it has been designed. Aircraft targets may operate at great altitudes even though manoeuvrability then becomes less. In such conditions, due to the more sluggish dynamic behaviour of the target, the target signal will be concentrated more at the lower frequencies. If the spectral density of the noise is increased around the frequency of response of the missile system, then the noise input to the missile guidance system will make greater demands for lateral acceleration. Thus, although the manoeuvrability of the target is decreased, there may still be a requirement for considerable lateral acceleration in the missile. This can only be achieved at great altitude by an increase in incidence of the lifting surfaces. In general, this will increase the non-linearity of the missile system and so accentuate the problems of guidance and control.

These problems are directly related to the aerodynamic form of the missile, which may have cruciform fixed wings or a pair of fixed wings in the so-called twist and steer configuration. Control surfaces may be situated near the nose or tail of the missile; instead of separate control surfaces moving wings may be employed. With any of these configurations there may be, in addition, stabilizing surfaces. Each of these configurations has advantages and disadvantages for high altitude operation at high incidence. In general a configuration giving least non-linearity and minimum cross-coupling will be preferred in order to ease the guidance and control problems. At the same time it is obviously necessary to design control systems which will, as far as possible, take into account the aerodynamic non-linearities. It may be noted that many of the problems of the guided missile at high altitudes are also, of course, shared by highly manoeuvrable piloted fighter aircraft operating at similar speed and height brackets.

In homing missiles, for example, difficulties arise due to radome aberration. This results in incorrect measurement of sight line angular rates and thus produces errors in the basic homing navigational data. By using a missile with moving wings, the angle between missile body axis and sight line remains relatively steady. This limits the apparent excursions of the sight line relative to the radome and so reduces the spurious rate signals caused by aberration. However, the incidence required on the wings is greater, since no use is being made of body lift, and the engineering problems in providing moving wings are more severe.

It is evident that adequate solutions of the problems can be obtained by suitable compromises between the various aerodynamic configurations, the control system and the method of obtaining target information. Care must, however, be taken to avoid too great a complexity, since this is likely to reduce the reliability of the system in operation.

3. GUIDANCE FOR THE BALLISTIC MISSILE AND SATELLITE VEHICLE

3.1 *Basic Principles*

The problems associated with guidance for ballistic missiles are different in kind from the anti-aircraft missiles, although, evidently, considerable use must still be made of the theory of servo-mechanisms. The ballistic missile spends most of its flight outside the atmosphere, which becomes a disturbing influence rather than a necessity as for the aircraft and the anti-aircraft missile. The atmosphere introduces disturbing forces during the launch and powered phase and at re-entry. In addition, for re-entry at high velocity, problems of kinetic heating must be solved. It is possible, however, to make use of aerodynamic forces during re-entry to control the flight path of the re-entry vehicle.

The target of a ballistic missile is not subject to manoeuvre and, for a satellite vehicle, the desired orbit for the satellite is known. The method of guidance employed depends on the constraints that can be put on the missile trajectory. Thus if the trajectory of the powered phase is completely predetermined with the motor being cut off at a predetermined missile position and velocity, the constraints on the missile are then most severe in following a fixed flight path and in requiring thrust control to arrive at each point on the trajectory at the appointed time. The minimum constraint will be achieved by omitting thrust control and cutting off the motors when the velocity vector is such that a free ballistic trajectory will take the vehicle to the target. At each point in space there is an infinity of velocity vectors which will take the vehicle through the target. Each of these velocity vectors represents an ellipse for the trajectory with the centre of the Earth as one focus. One of these velocity vectors has a minimum magnitude, representing the maximum range for a given missile. Some criterion must be decided to choose a particular required velocity vector for each point in space. The guidance method is then to measure the achieved velocity of the missile and compare it with the required velocity at the particular point and then to apply the missile acceleration in such a direction to reduce the difference to zero. There are evidently a number of variations on this scheme both in the criterion employed and the possible prediction of missile velocity from a series of measured values.

Between the two extremes requiring maximum and minimum constraints on the missile are a large number of possible methods, involving also more or less computational facilities to obtain the required signals to the control system. Thus it may be decided to use linear approximations for the equations to be solved to simplify the computation. Such approximations will define a volume around the nominal cut-off point within which the desired accuracy can be achieved. This will increase the constraints on the missile but reduce the complexity and size of the computer. The final solution will depend on whether the computer is on the ground or carried in the missile; this in turn will depend on how the measurements of missile position and velocity are to be made. Before considering these, it is useful to study the trajectory in a little more detail.

3.2 *The Ballistic Missile and Satellite Vehicle Trajectories*

The error at impact of a ballistic missile can be related to errors in position and velocity at earlier points along the ballistic trajectory. Over the whole trajectory, errors in position maintain a more or less constant importance whereas the effect of velocity errors varies roughly in proportion to the distance from the target. Thus near to the end of the trajectory, an error in position is more important compared with an error in velocity, whereas near the beginning errors in velocity assume the greater relative importance. This is illustrated in Figs. 8 and 9.

Since guidance is terminated most conveniently soon after motor cut-off, on all but short-range ballistic missiles velocity errors predominate. The effect of errors in components of velocity can be assessed approximately by transferring the error to the impact point and multiplying the

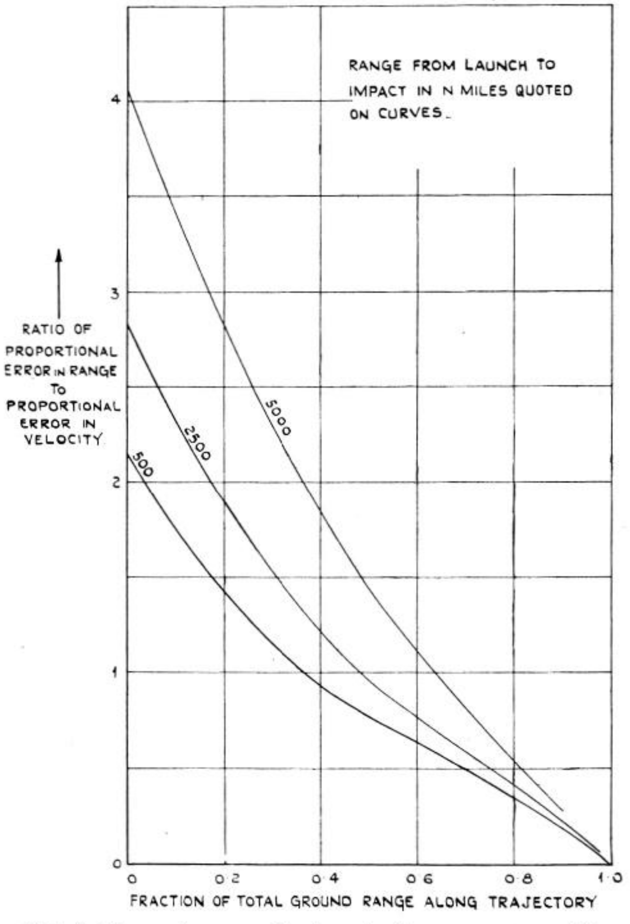


FIG. 8. Errors in range due to velocity error at cut-off.

magnitude by the missile time of flight. This is not strictly correct because of the variation of the gravitational field between the various trajectories. If the directions of the trajectory at cut-off and at impact are at right angles, an error in the velocity direction lying in the vertical plane of the trajectory has a negligible effect on the error at impact. The effect of this error is chiefly on the time of arrival at impact. By contrast, error in the magnitude of the speed at cut-off causes a large impact error.

For the satellite vehicle the conditions are somewhat different since the final launching of the satellite occurs at the top of the ballistic trajectory. If it is required that the satellite orbit is circular at a given height above the Earth, then it is necessary to control the height of the initial ballistic trajectory. Thus the direction in which velocity errors have least

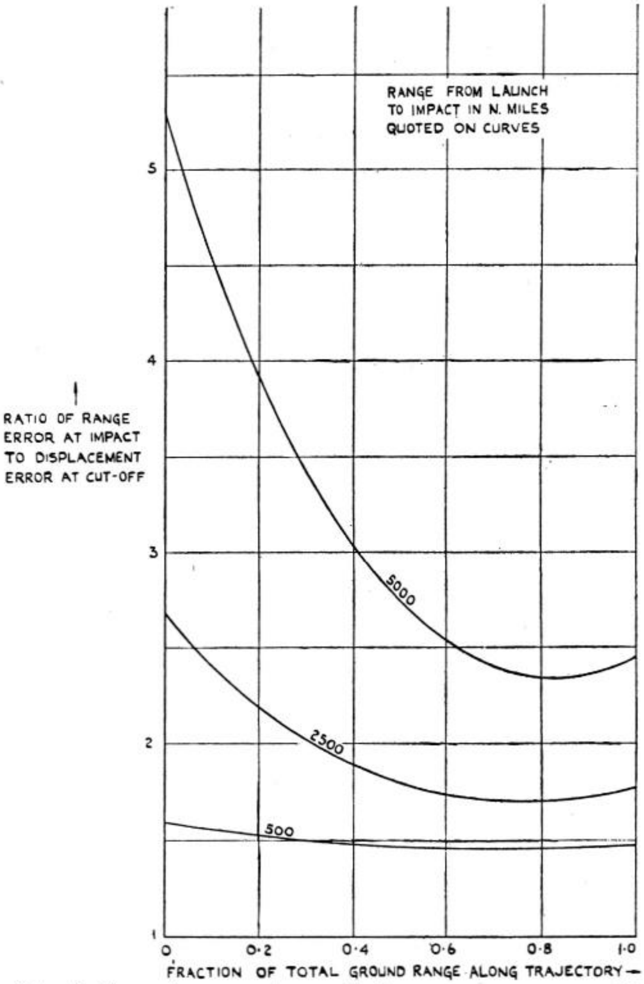
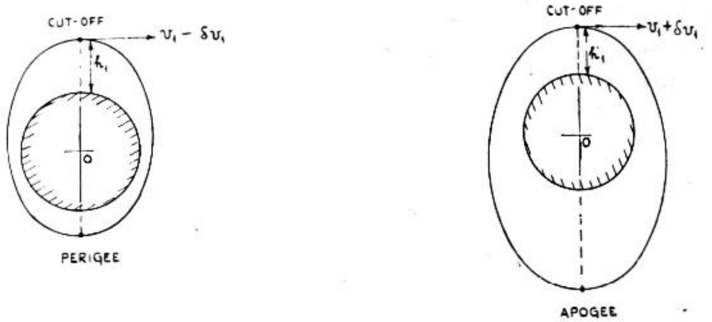
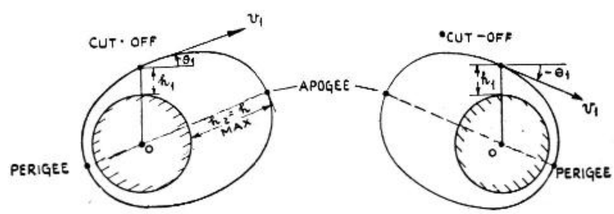


FIG. 9. Errors in range due to displacement error at cut-off.

effect is parallel to the trajectory at its maximum height. An error in this direction alters the distance from launch at which maximum height occurs but its value is not altered appreciably. The missile velocity at maximum height will then be different and this affects the additional velocity required to put the satellite into its orbit. The final thrust at the maximum height must be carefully controlled in direction and length of time for which it is applied if a near circular orbit is required, as, for example, for a recon-



(a) Final cut-off speed too small. (b) Final cut-off speed too large.



(c) Final cut-off speed inclined upwards. (d) Final cut-off speed inclined downwards.

FIG. 10. Circular satellite trajectory.

naissance satellite. The effects are illustrated in Fig. 10; the orbits shown exaggerate the departure from the desired circular orbit.

The requirements for guidance, in terms of the length of trajectory over which it is applied, depend on these trajectory considerations. In addition the requirements are affected by the length of trajectory under conditions of motor thrust. If further guidance is required after cut-off of the main motor thrust then subsidiary motors must be employed. Evidently, if subsidiary motors are employed, there is a compromise for highest accuracy between the effect of different errors along the trajectory, the actual errors in measurement along the trajectory and the penalties of subsidiary motors of different types in terms of thrust and length of time for which it is applied. The length of time for which motor thrust occurs, assuming it is not subject to thrust control, is dependent on the total weight of the vehicle at launch. The thrust must be just greater than the total weight, which will then decrease at a constant rate during flight until the fuel is finished. At the end of powered flight the acceleration will be very high, e.g. 10–20 g. The addition of subsidiary motors for guidance will add to the complexity of the missile and reduce its range performance. Hence it is desirable to complete guidance and cut-off the motors near to actual burn out. Essentially this means completion of guidance during the initial part of the trajectory and emphasizes the importance of velocity measurement in carrying out the guidance.

3.3 *Radar Methods of Measurements*

Two obvious and distinct techniques of measurement applicable to ballistic missile guidance are by means of radar and inertia navigation. The radar method uses measurement from the ground, computation of position and velocity, comparison with desired position and velocity and computation of a suitable demand to the control system of the missile. This demand is transmitted to the missile over a radio link. The standard radar measurements are of range and angle and the rates of change of these quantities. The measured values are subject to noise, both internal (receiver and servo-mechanisms noise) and external (fluctuations caused by variations in the propagating medium). Knowledge of the frequency spectrum of the noise together with the frequency spectrum of the missile motion (due to random variations in magnitude and direction of thrust) enables an optimum filter to be designed for the guidance system. In practice complete knowledge may not be available so that a best choice is made, taking into account possible variations in the frequency spectra and studying limiting cases by means of simulators. As with the anti-aircraft missile a process of theoretical studies, simulator work and flight trials is undertaken. An additional feature of the radar method is the complexity of the computation, partly due to the necessity for co-ordinate resolution and partly to the guidance equations which require solution. To obtain sufficient accuracy a digital computer is required and a finite time is taken in the computation. Thus the computed demands for the missile will lag in time. This can be overcome by predicting with the computer over its computation time and by making this a constant time interval. Thus there is a series of measurements of position and velocity, which are weighted according to their staleness, and used to predict the present position and velocity. The greater the prediction interval the more the noise is accentuated. Hence it may be preferable to predict over a shorter time interval, the lag in the data being less serious than the additional noise introduced.

If sufficient accuracy cannot be achieved during the main motor phase, then, as mentioned earlier, subsidiary motors can be used. These have very much lower thrust so that departures from a ballistic trajectory are small and the noise on the missile motion is much reduced. Thus smoothing of observations and prediction becomes more accurate. In addition the accuracy of cut-off of the small motors in terms of missile velocity is very much greater.

3.4 *Inertial Methods of Measurement*

The use of an inertial navigator for measurement introduces different problems in terms of accuracy. Essentially the inertial navigator consists of a platform, stabilized by means of gyroscopes with accelerometers mounted on it (Fig. 11). By contrast Fig. 12 shows an instrument still used

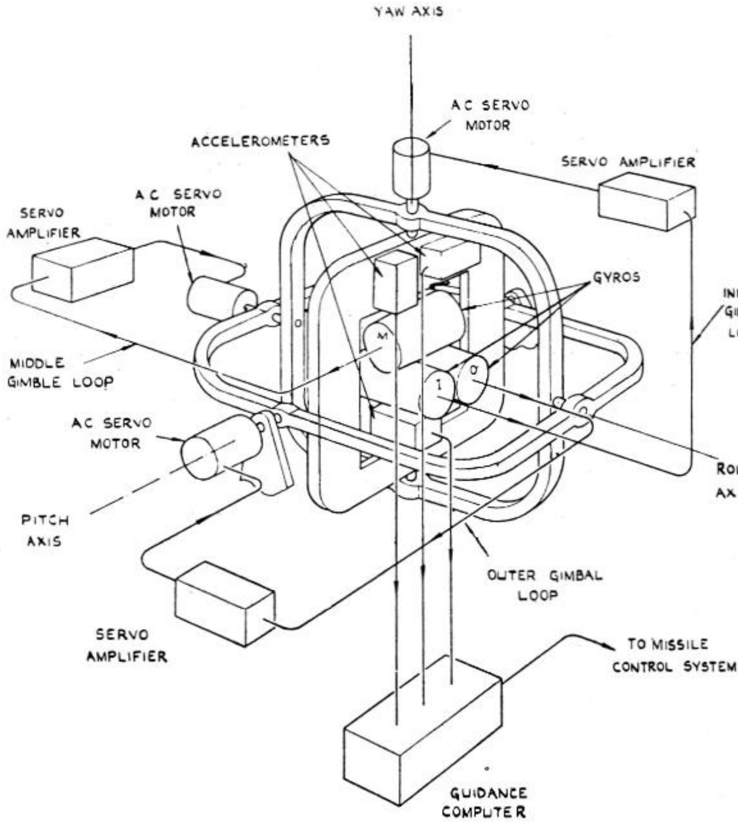


FIG. 11. Diagram of stable platform. Three-axis gyro-stabilized platform mounting three accelerators.

by mariners in the sixteenth century. Errors of measurement occur due to bias, scale error and noise in the output from the accelerometers and to variations in the orientation of the stable platform. The output of the accelerometers must be integrated to give velocity and position and errors can be introduced by the integrators. The effects of noise are small, compared with effects such as drift of the gyroscopes with time and with acceleration; due to this the inertia navigator is most accurate over short intervals of time and under low accelerations. With the ballistic missile the time interval of operation is short but the acceleration is high. The error due to drift is largely a function of the final velocity assuming the drift is proportional to acceleration. The final velocity is necessary in

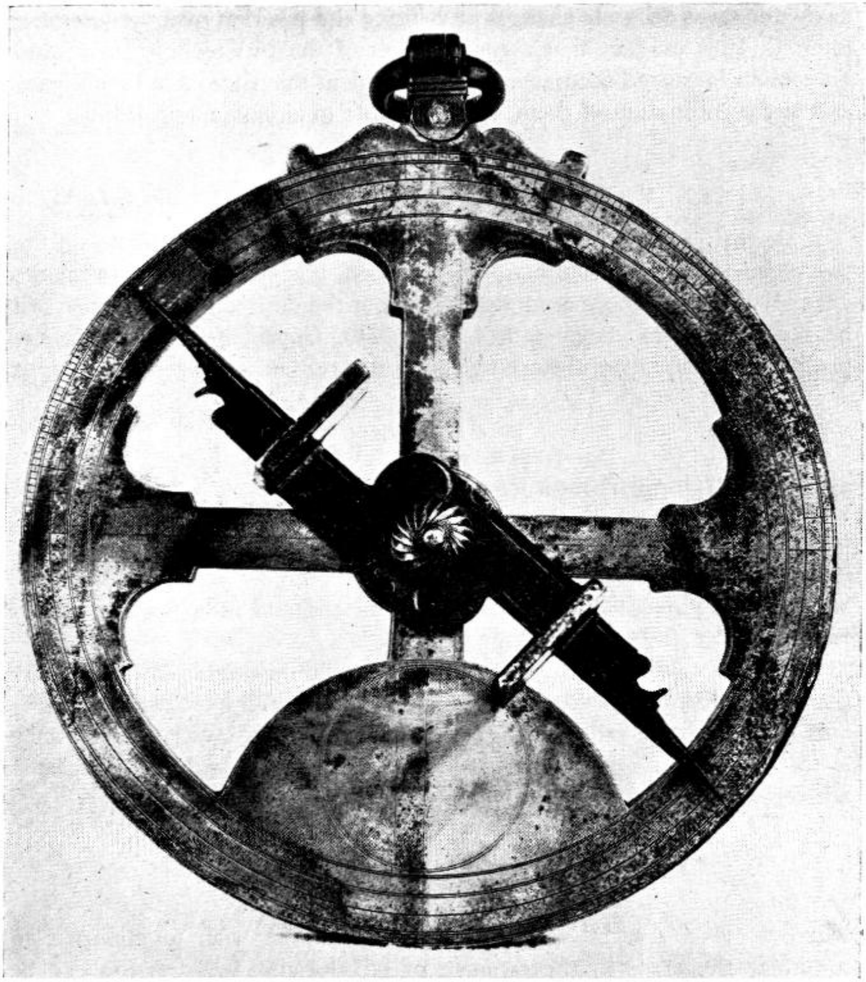


FIG. 12. The Mariner's Astrolabe 1588. (Copyright National Maritime Museum, Greenwich.)

order to obtain the missile performance; hence to a first order no improvement can be obtained by alteration of the acceleration programme of the missile.

It is evident, however, that the incremental accuracy of the inertia navigator is very high and information is produced with a negligible time lag.

The two methods of measurement, radar and inertia navigator, are seen to be complementary in that the long-term accuracy of the radar system and the short-term accuracy of the inertia navigator are both high. Thus the radar gives accurate measurements with a time lag and the inertia navigator gives accurate changes in velocity and position over a short time interval. It is evident that a combination of the two techniques should give much increased accuracy. It is also evident that these two techniques, and any combination of them, are applicable to aircraft navigation.

4. MIXED RADIO-INERTIAL NAVIGATION SYSTEM

If, as in the radio inertial system just cited, the random errors on the two channels of information are uncorrelated, it is possible to combine the channels to give a more accurate estimate of the desired quantity than can be obtained from either source separately. Consider for example the problem of estimating the true velocity v given an accelerometer output

$$A=\dot{v}+n_A$$

and a radio (Doppler) measure of velocity

$$R=v+n_R,$$

where n_A, n_R are random errors in the inertial and radio measurements respectively.

The way in which these two estimates should be combined depends on the information available about v, n_A and n_R. If nothing is known about v, and only the r.m.s. values of the errors are given, the best that can be done is to form the weighted average of the radio signal and the integrated accelerometer output. This gives

$$k\int(\dot{v}+n_A)\mathrm{d}t+(1-k)(v+n_R),$$

where k, $1-k$ are the weighting factors; and we wish to choose k to minimize the error in the estimate of v. The above expression can be re-arranged as

$$v+k\int n_A\mathrm{d}t+(1-k)n_R;$$

the error is therefore

$$k\int n_A\mathrm{d}t+(1-k)n_R,$$

and the mean square error σ^2 is given by

$$\sigma^2=k^2\sigma_v{}^2+(1-k)^2\sigma_R{}^2,$$

where $\sigma_v{}^2$ is the mean square value of the *integrated* accelerometer error and $\sigma_R{}^2$ that of the Doppler error.

The value of k which minimizes the resulting error σ^2 is easily found to be

$$k=\sigma_R^2/(\sigma_R^2+\sigma_v^2),$$

for which value

$$\sigma^2=\sigma_R^2\sigma_v^2/(\sigma_R^2+\sigma_v^2).$$

If only one source of information were available the error would be σ_v^2 (using the accelerometer) or σ_R^2 (using the radio data), both of which are larger than σ^2, so that a reduction in the error by combining the two sources can be guaranteed—provided that the weighting factors are correctly chosen. This latter proviso is important, since it is easily possible to increase the error with an arbitrary weighting factor. If, for example, we take the unweighted average ($k=\frac{1}{2}$) the error is

$$\sigma^2 = \tfrac{1}{4}(\sigma_v^2 + \sigma_R^2)$$

which would be greater than the smaller of σ_v^2 and σ_R^2.

A further improvement can be obtained if the frequency distributions of the random errors are known; to this end the weighting factors k, $1-k$ are replaced by time-dependent weighting functions, chosen to reject those frequencies at which the noise is troublesome. We can represent this by a general operator $F_A(D)(D=\mathrm{d}/\mathrm{d}t)$ acting on the accelerometer output, and $F_R(D)$ on the radio signal, before the two are combined. The resulting quantity is

$$F_A(D)(\dot{v} + n_A) + F_R(D)(v + n_R)$$

or

$$[DF_A(D) + F_R(D)]v + F_A(D)n_A + F_R(D)n_R.$$

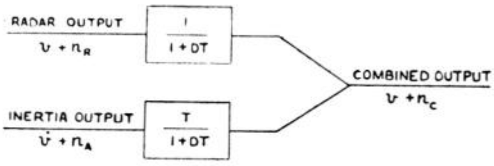
If nothing is known about the behaviour of the true velocity v it is not possible to assess the error induced by any operation on v; we must, therefore, arrange that

$$DF_A(D) + F_R(D) = 1$$

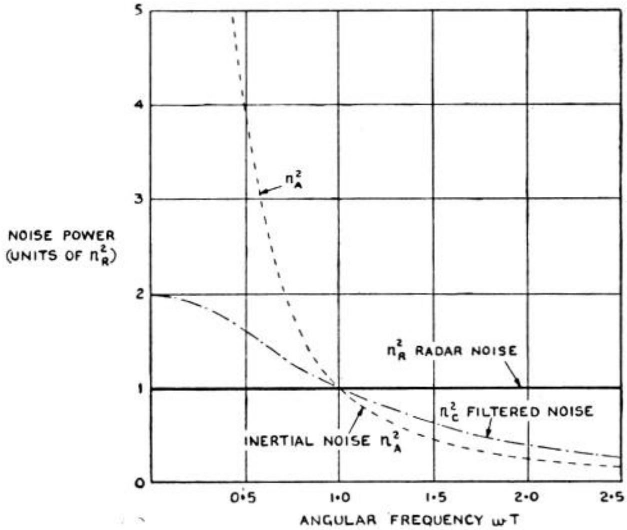
in which case the expression becomes

$$v + \frac{1 - F_R(D)}{D}\, n_A + F_R(D)n_R$$

and the error is composed of the second and third terms. It is possible now to choose $F_R(D)$ (and hence $F_A(D)$) to minimize the mean square error, using the method of Wiener; the resulting expressions for $F_R(D)$ and $F_A(D)$ are in terms of the spectral densities of the random errors n_A and n_R. These functions must, of course, be known or assumed in order to define the operators explicitly.



(a) BLOCK SCHEMATIC



(b) NOISE POWER SPECTRA BEFORE AND AFTER FILTERING

FIG. 13. Radar-inertial mixing.

As a simple example, let us assume "white noise" for both the Doppler and accelerometer measurements, of spectral densities k_R^2 and k_A^2 respectively (see Fig. 13). The optimal operators are then

$$F_R(D) = \frac{1}{1 + DT}$$

$$F_A(D) = \frac{T}{1 + DT}$$

where $T = k_R/k_A$.

The physical embodiment of $F_R(D)$ could be a simple R.C. filter of time constant T; and for $F_A(D)$ the same filter followed by a gain T.

With these optimal filters the mean square error is

$$\sigma^2 = \pi k_R k_A,$$

which is smaller than could be obtained from either system separately.

It will be noted that the accelerometer output is not integrated; the operator $T/(1+TD)$ only approximates to integration for large T—i.e. when the accelerometer information is very good (k_A small) or the radio information very poor (k_R large) or both. If there is no accelerometer error ($k_A=0$) T becomes infinite, so that there is no contribution from the radio measurement and the output is simply the integrated accelerometer signal. At the other extreme, with perfect radio information ($k_R=0$) the radio filter becomes unity and zero weight is given to the accelerometer.

The use of the correct operators is of great importance; it is not sufficient to choose them on some arbitrary basis, such as "smoothing out high-frequency noise", since the resulting error in the combined system may well exceed the error of either source taken on its own. On the other hand, the definition of the optimal operators depends on a knowledge of the spectral densities of the noise from each source—not easy functions to establish. Indeed, the noise signals may vary in such a way that they can only be regarded as stationary processes over relatively short periods—i.e. the spectral densities (if they can be so defined) change with time. In this situation it would be necessary to make a continuous adjustment of the operators, using, for example, the self-optimizing technique mentioned above.

The example above has not used any information about the variable being estimated—the true velocity. In the absence of such information, the operators so deduced represent the best that can be done in this situation. If, however, some characteristics of the variation of velocity with time are known *a priori*, the accuracy of the estimate can be further improved. The most useful additional information is again the spectral density functions; if this is known the restriction

$$DF_A(D) + F_R(D) = 1$$

is removed and optimization proceeds without this constraint. The resulting operators are then functions of the spectral densities of v, n_A and n_R. As would be expected, the r.m.s. error is smaller than that obtained when no information about v is available.

Although the above is a particular example, it will be obvious that the method is quite general, and can be applied in a variety of situations. Two or more sources can always be combined in such a way that the final error is reduced; this is true even when the random errors from each source have the same frequency distribution, provided they are uncorrelated. The improvement obtained depends on how much is known about the random errors and the true signal, and this knowledge also defines the optimal operators. If operators other than these are used—or if the actual statistics differ from those assumed or estimated—the results may well be worse than that achieved with either system used alone.

5. GUIDANCE TO THE SATELLITE SPACE STATION

5.1 *General Principles*

The problems of navigation, guidance and control for conventional aircraft and missile requirements are reasonably well understood, although solutions to these problems in terms of equipment are by no means finalized. It is of interest to speculate on the new problems of space flight. Earth satellites have been launched and have entered predetermined orbits with moderate accuracy; but the stage has not yet been reached of defining the orbit, prior to launch, in an accurate manner in terms of height and eccentricity. Some of the main problems arising are known together with theoretical techniques for producing accurate systems of guidance and control. Performance studies have been made of rockets required to reach the Moon and these studies have involved consideration of possible trajectories and the navigation accuracies required. Essentially such studies have been an extension of the techniques of ballistic missiles.

Looking further into the future many proposals have been made, from the use of the principles of ballistic missiles for manned flight over the Earth's surface to the establishment of large Earth satellites as space stations and finally to interplanetary travel. In principle all these projects are feasible and it is sensible that performance studies should be made to assess the major problems involved. In practice it is unlikely that much success will be achieved unless they are studied as completely integrated systems; in particular, guided missile experience has shown that the requirements of guidance and control are bound to have marked repercussions on the vehicle structure and propulsion system design. To a large extent, guidance and control problems have been ignored by the space travel enthusiasts, the assumption being made that suitable techniques will be developed at the right time to fit into the proposed designs of vehicles. It is suggested that this is the wrong approach; instead the vehicle and its propulsion should be designed around the payload and the guidance and control system.

Some idea of the problems involved in space navigation can be gained by considering the simple case of delivering a payload from the Earth to an existing Earth satellite. It is desirable that the payload should be put into the satellite orbit on an optimum trajectory since this gives maximum payload for a given missile size. In addition any adjustments required to bring the payload to the satellite will require the expenditure of energy to cause acceleration or deceleration; this means the use of rocket thrust and consumption of fuel. The weight of this fuel must be kept to a minimum since it detracts directly from the payload. Evidently there is a compromise to be made here between guidance during the initial boost phase, with departures from the optimum trajectory, to bring the payload as near as possible to the satellite, and subsequent guidance to coincidence with the satellite. Both of these will result in reduction of payload from the theoretical maximum. The magnitude of departure from the optimum trajectory will depend on errors in the time of launch, variations in the thrust programme, effect of winds in the early part of the boost phase and small differences in performance between the real missile and the design predictions. These effects could be studied and an assessment made of accuracy of arrival into the desired orbit in terms of payload reduction. For a given accuracy of arrival the requirements in terms of fuel to bring the payload and satellite into coincidence could then be assessed. By this method some estimates could be made of the overall performance requirements. To appreciate the navigation requirements we must study the characteristics of typical satellite orbits. In the following section a few particular cases are chosen to illustrate the unusual problems which may arise; it is assumed that the payload has been launched into an orbit approximating that of the satellite with which a rendezvous in space is planned.

5.2 *Earth Satellite Orbits*

1. *Satellite and payload in the same circular orbit but separated in position*—By giving to the payload a velocity increment along its trajectory at some point, this point becomes the perigee of the resulting elliptical orbit. The period of rotation about the Earth is then increased so that the angular displacement of the satellite and payload alters. By choosing the correct change in velocity the satellite and payload could be brought together at the perigee of the payload orbit. At this point it would be necessary to produce the opposite velocity change for them to stay together.

For example, if the payload is one minute ahead of the satellite in the orbit (corresponding to an angular separation subtended at the Earth centre of roughly 4°), then to obtain coincidence after one revolution requires an increase of one minute in the payload orbital time. The orbital time is given by

$$T = \frac{2\pi}{R}\sqrt{\frac{a^3}{g}}$$

where R = radius of the Earth
a = semi-major axis of the orbit
g = acceleration due to gravity

$$\mathrm{d}T = \frac{3\pi}{R}\sqrt{\frac{a}{g}}\mathrm{d}a$$

Very roughly for near satellites, taking $a=R$

$$\mathrm{d}T\,(\text{sec}) \simeq 2\ \mathrm{d}a\,(\text{miles})$$

Thus the one minute can be lost by an increase of height in the payload (at apogee) of about 60 miles. This corresponds to an increase in velocity of the payload by about 100 ft/sec. It is noteworthy that the payload needs to increase speed to approach the satellite station from in front; that is the correcting impulse is directed away from the target. It should be noted that a 1 ft/sec error will result in a separation of payload and satellite of about 3 miles at the perigee of the payload orbit; but they will approach each other more closely about 10° from the perigee. Thus it would probably be necessary to remove major errors by an alteration in the orbital period and the remaining errors by some more continuous form of guidance.

It is evident that the major error can be removed by means of a very small increase in velocity, provided the time to arrive at coincidence of payload and satellite can be large, i.e. after a large number of revolutions. With this is associated a need for greater absolute accuracy. However, very slow interceptions are attractive only on the simple theoretical treatment; in practice it would be necessary to take into account perturbations of the orbits.

2. *Satellite and payload in different circular coplanar orbits*—The orbit of the payload can be increased or decreased by applying thrust along the trajectory. If an increase in velocity is made then the elliptical trajectory can be made tangential at its apogee to the satellite orbit and a further suitable increase of velocity at the apogee will give the same orbit as the satellite.

3. *Satellite and payload in different non-coplanar orbits*—Rotation of the payload orbit is required and this necessitates a thrust component perpendicular to the orbital plane in order to change the direction of the velocity vector. However, it must be remembered that, in any case, due to the oblateness of the Earth, both orbital planes rotate in space. The rate of rotation is

$$10{\cdot}00\left(\frac{R}{\bar{r}}\right)^2\left(\frac{R}{a}\right)^{1\cdot5} \cos\alpha \text{ degrees per day}$$

The major axis also rotates in each orbital plane at a rate given by

$$5{\cdot}00\left(\frac{R}{\bar{r}}\right)^2\left(\frac{R}{a}\right)^{1\cdot5} (5\cos^2\alpha - 1) \text{ degrees per day}$$

where $\bar{r}$ is the harmonic mean distance from the centre of the Earth, a the semi-major axis and α the inclination to the equator. Thus, in general, both the ellipse and its orbital plane will be rotating in space, the rate depending on $\bar{r}$, a and α. Evidently knowledge of these rotations is essential for computing the orbit adjustments necessary to bring the payload and satellite orbits into the same plane and for achieving coincidence of the semi-major axes.

Even for a simple control force the problems involved have not received serious attention. It is not suggested that the above adds anything to the solution of these problems, it simply points to their existence. The solution to the problem of interception with minimum energy consumption in an inverse square law field of force would not appear insoluble. To carry out the interception but arrive with the same velocity as the satellite appears more difficult. In addition it will be necessary to do the analysis for the true gravitational field, taking into account the resulting perturbations of the satellite orbits.

A major problem will be the provision of reference directions in the payload so that, even if guidance is carried out by command from the ground, thrust can be applied in the required direction. After major correction, some form of terminal guidance will probably be necessary to bring the payload alongside the satellite without excessive expenditure of fuel.

It is hoped that a rough indication of the magnitude of the problems has been given and that performance studies on space stations and interplanetary travel are not regarded as more than a modest beginning of the extensive and painstaking exploration of the upper atmosphere and near terrestial interplanetary space which lies ahead.

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DISCUSSION

A. Stratton*: I would like to add a few words by way of example to illustrate how inertial navigation techniques may be applied in aeronautics without the need for the precise components that are required for full inertial navigation. The case I will consider is the mixing of inertial information with that from a radio altimeter to provide instantaneous vertical velocity for landing, by means of the arrangement of Fig. 13. In this case the filter for the "radar output" would be modified to $D/1+DT$.

The main "noise" from the inertial system will be any misalignment of the vertical accelerometer from the vertical, resulting in a component of the aircraft's horizontal acceleration being detected. Taking, for example, an aircraft banked at

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45°, if the misalignment of the accelerometer is 0·2° from the vertical in a plane containing the vertical and the wings, then an acceleration error of approximately 0·1 ft/sec^2 will obtain. If the time constant T is 30 sec this will result in a velocity error increasing to a steady value of 3 ft/sec with a time constant of 30 sec from the application of the turn.

The main noise from the radio altimeter on the other hand is likely to be discontinuous or rapid changes in measured height. Considering, for example, a step change in output Δh this will give an instantaneous error $\Delta h/T$, which decays with time constant T sec. A step change of 90 ft would therefore give a velocity error of 3 ft/sec with a time constant of 30 sec.

The vertical error of 0·2° which has been assumed in this example would correspond to a navigational error of 12 n.m. if it obtained in an inertial navigation system. It is evident, therefore, that considerable smoothing of radio information may be possible without requiring components of the accuracy that would be needed for pure inertial navigation.